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THEORY EXTENSION AND NUMERICAL EXPERIMENTS TO DETERMINE THE PROBABILITY OF EXCEEDANCE OF GIVEN LOAD LEVELS IN SPECIFIED TIMES

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Extensions to the theory of the "first encounter" problem are developed. Numerical experiments with actual time histories were conducted to establish the probability density distribution functions for the intervals between like crossings. Time histories considered include a random function, vertical acceleration records due to gust encounter, and vertical gust velocities of continuous atmospheric turbulence. It appears that the density distribution function for practical cases can be represented well by two

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20. ABSTRACT (cont.)

exponential terms (a single term applies to a time sequence of independent events). The associated curves giving the probability of encountering a given level of the phenomenon under consideration in a specified time are developed. Examples are given to show application in the gust loads encounter problem.

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INTRODUCTION

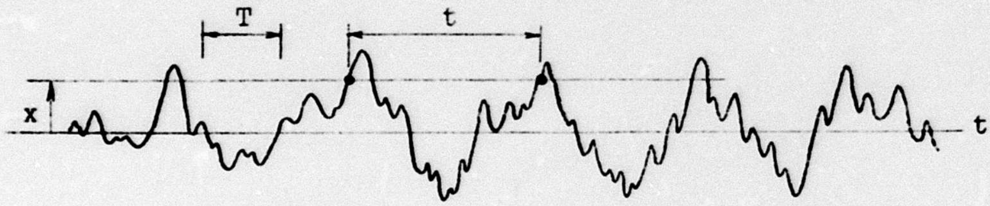
In the design of structures one of the problems that is of concern is the so-called first encounter problem. Thus, we wish to know in a given exposure time: what is the probability that a load of a given magnitude will be reached? We find this concern in a number of different applications. In aeronautics, one of the concerns is the encounter of large gust loads. In the design of buildings or large outdoor structures, we are concerned with wind loads and loads due to earthquakes (the probability that wind loads or earthquake loads of a given magnitude will be reached in n years).

Reference 1 gives a general theoretical treatment of this probability problem. Key in the development is a probability density distribution relating to the interval of time between the encounter of like load levels. For a sine wave the interval of time between the crossing of like levels is constant; the probability density function for the interval between crossings of a given magnitude is thus a "spike" or Dirac function, located at the period of the wave. For a random function the interval of time between crossings of like levels is variable. The probability density function associated with intervals between like crossings depends on the nature of the random function under consideration. The probability of encounter of a given level in a time T is thus also dependent on the nature of the function being considered.

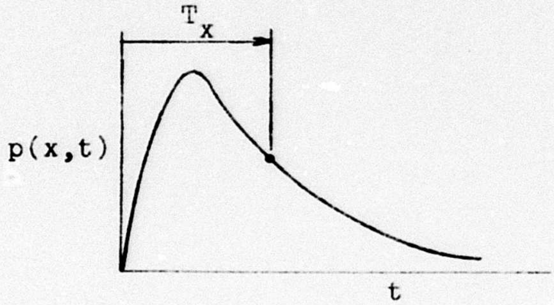
In this report, some extensions to the theoretical developments given in reference 1 are made. In addition, as a main goal, an experimental type study is made of several types of functions to establish what the probability density functions for intervals between like crossings are like in real situations. Three different type functions are considered: 1) a random function as obtained from a random noise generator, 2) "gust" acceleration records as obtained from an airplane encountering continuous atmospheric turbulence, and 3) deduced gust velocity records of continuous turbulence.

THEORY EXTENSION

Consider the random function shown in the following sketch:



We are interested in the probability that the level x will be experienced in the time T. In general, the interval t between like crossings of x is variable. If we consider all the t values that are indicated in a record of large length, we can form a probability density distribution of these intervals, which may appear as follows



In this sketch, T_x denotes the average of all the t values considered. The p function has the following properties

$$\int_0^\infty p(x,t)dt = 1 \qquad (1)$$

$$\int_0^\infty tp(x,t)dt = T_x \qquad (2)$$

Reference 1 develops the theory showing that, in terms of the function $p(x,t)$, the probability of encountering x in a time T is given by the equation

$$P(x,T) = 1 - \frac{1}{T_x} \int_T^{\infty} (t - T)p(x,t)dt \qquad (3)$$

It is to be noted that this equation applies to all stationary functions whether random or not. The precise form of p depends on the nature of the function.

As mentioned in the introduction, p for a sine wave is a Dirac function at the period of the wave. In this case, equation (3) reduces to

$$\left.\begin{aligned} P(x,T) &= \frac{T}{T_x} \quad ; \quad && T \leq T_x \\ &= 1 && T \geq T_x \end{aligned}\right\} x \leq x_m \qquad (4)$$

where T_x is the period of the wave, and x_m is the sine wave amplitude.

If p is given by

$$p = \frac{1}{T_x} e^{-\frac{T}{T_x}} \qquad (5)$$

equation (3) yields

$$P(x,T) = 1 - e^{-\frac{T}{T_x}} \qquad (6)$$

which is associated with a Poisson distribution. Equations (5) and (6) apply to a function, or series of events, in which each event is independent of all previous events. We are interested in what p is like, and in turn P, for phenomenon such as the gust encounter of airplanes.

Some extensions to the theory of reference 1 follow. Successive derivatives of equation (3) are

$$\frac{dP}{dT} = \frac{1}{T_x} \int_T^{\infty} p(x,t)dt \qquad (7)$$

$$\frac{d^2P}{dT^2} = -\frac{1}{T_x} p(x,T) \qquad (8)$$

Boundary conditions indicated are as follows. Equation (3) indicates

$$P(x,0) = 0 \tag{9}$$

$$P(x,\infty) = 1 \tag{10}$$

Equations (1) and (7) indicate

$$\left.\frac{dP}{dT}\right)_{T=0} = \frac{1}{T_x} \tag{11}$$

Two ways are thus available for establishing the probability function P from a given p. One way is to make direct use of equation (3). A second way is to use equation (8), integrate, and, in the process, make use of boundary conditions (9) and (11).

In reference 1, two general forms of p were assumed, and associated results for P were established. We conclude this section by presenting two more forms of p that are of possible interest. The first assumes that the p function is represented by two exponential functions as follows:

$$T_x p(x,t) = \frac{n+1}{n-1}\left[e^{-\left(1+\frac{1}{n}\right)\frac{t}{T_x}} - e^{-(1+n)\frac{t}{T_x}}\right] \tag{12}$$

where n is a "family" variable; note, this representation of p yields $p(x,0) = 0$. The associated $P(x,T)$ curves, as established by either of the two approaches described, are given as follows:

$$P(x,T) = 1 - \frac{1}{n^2 - 1}\left[n^2 e^{-\left(1+\frac{1}{n}\right)\frac{T}{T_x}} - e^{-(1+n)\frac{T}{T_x}}\right] \tag{13}$$

Results for p and for P are shown in figures 1 and 2.

In the second case, we assume p is in the nature of a Rayleigh-type distribution, or

$$T_x p(x,t) = \frac{\pi}{2}\frac{t}{T_x} e^{-\frac{\pi}{4}\frac{t^2}{T_x^2}} \tag{14}$$

The derived result for P is

$$P(x,T) = \operatorname{erf}\left(\frac{\sqrt{\pi}}{2}\frac{T}{T_x}\right) \tag{15}$$

These functions are shown in figures 3 and 4, in comparison to the results given by equations (4), (5), and (6).

EXPERIMENTS WITH REAL FUNCTIONS

Studies to determine the actual nature of the p function do not appear to have been made. Some numerical experiments were made, therefore, to gain an insight as to their makeup. Three different type functions were examined:

1) A random function as obtained from a random number generator.
2) The vertical acceleration record as obtained from an airplane traversing continuous turbulence.
3) The measured vertical velocities of atmospheric turbulence.

Results obtained are described in the following sections.

Random function.- Typical p functions as deduced from a random function (in this case, a Gaussian-white-noise function) are shown in figure 5. Two curves are shown, one for a response level of $.25\sigma$, the other for a level of 1σ, where σ denotes the rms value of the function. The most striking aspect noted is the initial rise in the function to a peak and then the fall-off thereafter; this should be contrasted to the $n = \infty$ function shown in figure 1, which applies to a series of independent events. Further insight is gained if the p functions are plotted in semilog form, as shown in figure 6. We see that, after the peak, the function behaves in exponential fashion, thus indicating that, when the interval between crossings is greater than some value, events behave in independent manner, as might be expected. Figure 6 applies to crossings of a positive level, or crossings of a negative level. If we consider consecutive crossings, regardless of sign (see sketch in figure 7), then results as shown in figure 7 are obtained. The main difference to be noted in the results of figure 7 is that the T_x value is one-half the value of the 1σ results shown in figure 6, which of course should follow. A very definite exponential behavior beyond a t of about 3 seconds is noted in figure 7.

Vertical accelerations due to gusts.- The p function that was found by analyzing an actual vertical acceleration record obtained during an airplane gust encounter is shown in figure 8. The dashed line represents a fitted exponential function to the data at high t . We note, again, a rise in the function to form a peak at low values of t . The dotted curve and the dashed curve considered in combination correspond roughly to the $n = 6$ curve in figure 1. Note, it is because of the results shown in figures 5 and 8 that a development of the functions shown in figures 1 and 2 were made.

The fact that the p function shown in figure 8 does not behave in exponential fashion all the way to $t = 0$ has a physical interpretation. The $n = \infty$ curve in figure 1 applies to a sequence of completely independent events. For physical systems, however, successive response values within short intervals of time cannot be independent of one another because of the response characteristics of the system. Consider figure 9. The top sketch depicts a random input to a system under consideration. The second sketch denotes the impulse-response function of the system, here folded over as in the sense of the superposition or convolution integral. The bottom sketch denotes the response that is being developed (the integration of the product of the top and middle sketches). We note the contrast of considering short time intervals of response, compared to long time intervals. Consider, for example, the cluster of large peaks in the input in the region a . These peaks cannot come through in an independent way in the output because of the averaging out, or "smearing" out properties of the system response characteristics. Thus, level crossings in the response in the region of a (or b or c), are not strictly independent of one another. However, what happens in the region b is independent of the region a , because the system has lost all memory of what was taking place at a . In the same way, the response at c is independent of what has occurred at b or a . Thus, in reference to the p curves, such as shown in figure 8, it is plausible that the

results at the larger t values should behave in the manner of independent events (an exponential fall off) but that for low t's (which fall within the characteristic response time of the system), there should be a marked difference in comparison to the independent event results. Note, until a more thorough study is made, we must admit that the results shown in figure 5 suggest a possible anomaly relative to the discussion just given of figure 8. Figure 5 applies to a random sequence of numbers, as obtained from a random number generator; presumably each number generated should be independent of previous numbers, and thus we might expect that results should conform to the $n = \infty$ curve of figure 1. There is a marked drop-off near the origin, however, as is noted in the airplane response results given in figure 8; the reason why this drop also appears in the case of the random function is not clear.

Vertical gust velocities.- The vertical gust velocity record shown in figure 10 was analyzed for the intervals between level crossings. Results for the p function found are shown in figure 11. We note the tendency for the results to drop off near the origin, as with the acceleration results of figure 8. Reasoning along the lines used in connection with figure 9 should also apply to these results. The laws of fluid motion and the equation of continuity dictate that turbulence velocities in the proximity of one another cannot be independent of one another. We know that turbulence velocities are described by a correlation function; the turbulence velocities at one point are not independent of the velocities at another point unless the separation distance is large enough to make the correlation between the velocities at the two points essentially zero. This separation distance is generally on the order of the integral scale of the turbulence, which, for atmospheric turbulence, appears to be in the order of 500 to 1000 ft. Thus, if an airplane is traveling at 500 fps, velocity peaks occurring within a 1 to 2 second period are not necessarily independent of one another.

Unfortunately, it is not possible to infer the quantitative nature of the p function for atmospheric turbulence from figure 11, for two main reasons. One, the turbulence is not strictly stationary, as can be seen in figure 10. Second, the record length is not long enough. The experiment is in need of repeating with a record that is more stationary and which is considerably longer, so that statistical reliability and a quantitative measure of p can be obtained.

EXAMPLE PROBABILITY CONSIDERATIONS

In this section we show how specific probability results may be obtained. Examples are given for the situation of an airplane encountering atmospheric turbulence. For this situation, it has been found convenient to express load exceedance results in a form such as shown in figure 12, see references 2 and 3; in this figure, N denotes the number of upward crossings per second of the load level x, N_o is the number of upward crossings per second of the mean or 1-g level, and A is a structural response quantity which relates the rms value of the response x to the rms value of the gusts w by the relation

$$\sigma_x = A\sigma_w$$

Denote the value of the curves by $f(\frac{x}{A})$, and thus $N = N_o f(\frac{x}{A})$. Since N represents a statistical average, the average repeat time for a crossing is

$$T_x = \frac{1}{N}$$

which makes the abscissa in figure 2 appear as

$$\frac{T}{T_x} = TN_o f(\frac{x}{A}) \qquad (16)$$

This applies to either a positive gust encounter, or a negative encounter. If encounter is considered regardless of sign, the average repeat time would be one-half of T_x, and thus $\frac{T}{T_x}$ would be twice the value indicated by equation (16). It is of interest to note that equation (16) also applies to the number of times n the value of $\frac{x}{A}$ will be reached on the average in a time T; thus,

$$n = TN_o f(\frac{x}{A})$$

Consider now the following problem. What is the probability that an airplane will encounter a value of $\frac{x}{A} = +60$ fps in 10,000 hours of flight at 20,000 ft altitude, assuming that $N_o = 1$ and that the $n = 6$ curve of figure 2 applies. (Note

values of $\frac{x}{A}$ in the neighborhood of 50-60 correspond to structural design values due to gust encounter.) From figure 12 we find $f(60) = 1.5 \times 10^{-7}$, which gives

$$\frac{T}{T_x} = 10{,}000 \times 3600 \times 1 \times 1.5 \times 10^{-7}$$
$$= 5.4$$

From figure 2 we find $p = .9981$; this means that if the experiment were repeated 1000 times, one could expect at least one encounter of $\frac{x}{A} = 60$ in 998 of the cases, and 2 cases should be free of this encounter. Note, on the average, there would be 5.4 encounters of the 60 level during the 10,000 hours of flight.

Second example: establish the probability of encountering a $\frac{x}{A} = 50$ value during a one-hour flight at sea level in conditions which are 2.5 times as severe as on the average; take $N_o = 1$. The $f(\frac{x}{A})$ value for this case may be found from figure 12 by making two adjustments. The curves in this figure reflect automatically the proportion of time the airplane is in turbulence on the average. For the consideration of a continuous emersion in turbulence, the ordinates should be divided by the intercept at $\frac{x}{A} = 0$ (for the $h = 0$ altitude, this intercept is .7). The consideration of severities greater than average is taken into account by dividing the given $\frac{x}{A}$ value by the factor of increase, and then to use this newly found value for $\frac{x}{A}$, thus $\frac{x}{A} = \frac{50}{2.5} = 20$. For this example, f is found from figure 12 to be $f = .0013$. Equation (16) gives

$$\frac{T}{T_x} = 1 \times 3600 \times 1 \times .0013$$
$$= 4.68$$

Figure 2, $n = 6$, gives in turn, $P = .9956$. This example shows the hazards of operating in conditions which are known to be much more severe than the average, even for relatively short periods of time. Considerations of this type thus may be helpful in establishing whether or not certain missions should be flown.

CONCLUDING REMARKS

Extensions to the theory of the "first encounter" problem have been considered herein. Some numerical experiments were conducted with several time history functions to establish the nature of the probability density function that is associated with the time intervals between like crossings, and which is basic in the determination of the probability function that a certain level of the phenomenon under consideration will be reached in a given exposure time.

It is found that, for the functions studied, which includes realistic gust acceleration and vertical gust velocity time histories, the probability density function for the interval between like crossings behaves in exponential fashion for the larger intervals between crossings. This behavior is in agreement with that which applies for the case of a time sequence which is composed of a series of independent events. For the short intervals between crossings, the density function seems to depend on the type of time history being considered. The inference is that the density distribution function for intervals between crossings is dependent on the autocorrelation function of the time history. The fact that, at large time intervals, there is little or no correlation, seems synonymous with the fact that the events are independent. At short intervals, where correlation exists, the events are not independent.

Further study should be made of realistic gust velocity records having greater lengths than those available for the present study, so that the characteristic density distribution function for gust encounter can be established more reliably than herein.

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2. Houbolt, John C.: "Design Manual for Vertical Gusts Based on Power Spectral Techniques." Technical Report AFFDL-TR-70-106, December 1970.

3. Houbolt, John C.: "Updated Gust Design Values for Use with AFFDL-70-106." Technical Report AFFDL-TR-73-148, November 1973.

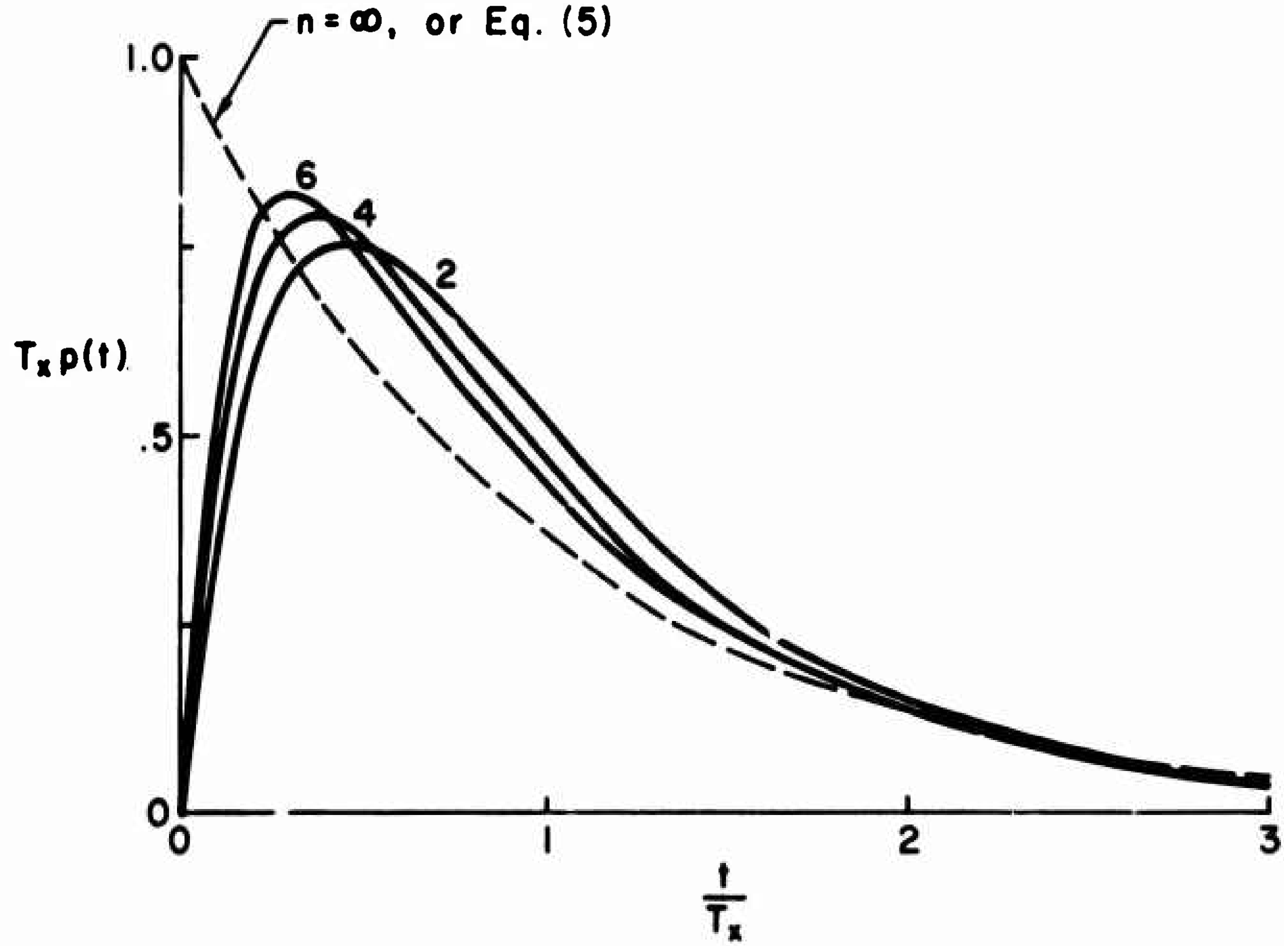

Fig. 1. Distribution Function for Repeat Time; Equation (12)

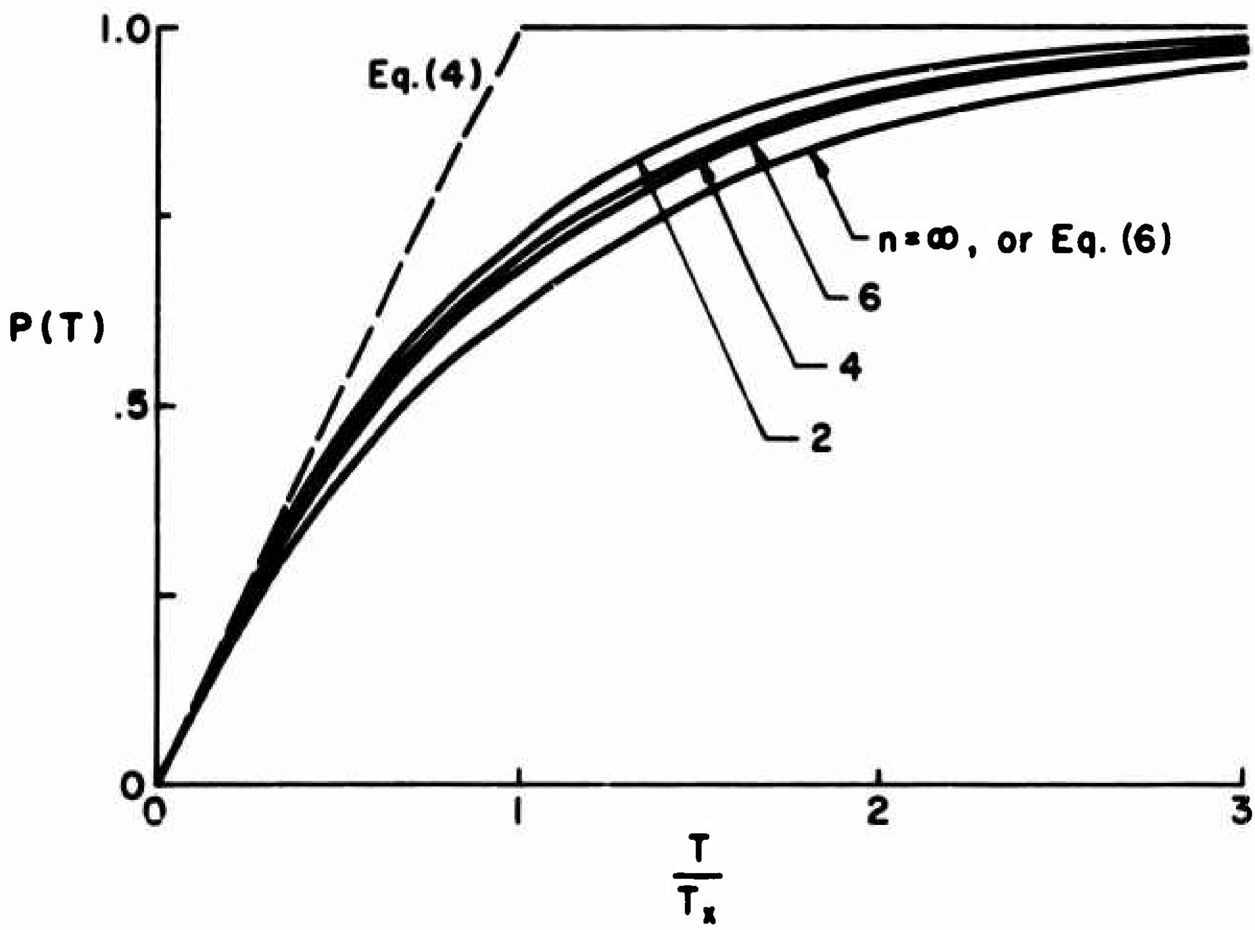

Fig. 2. Probability of Exceeding Load Level x in time T ; Equation (13)

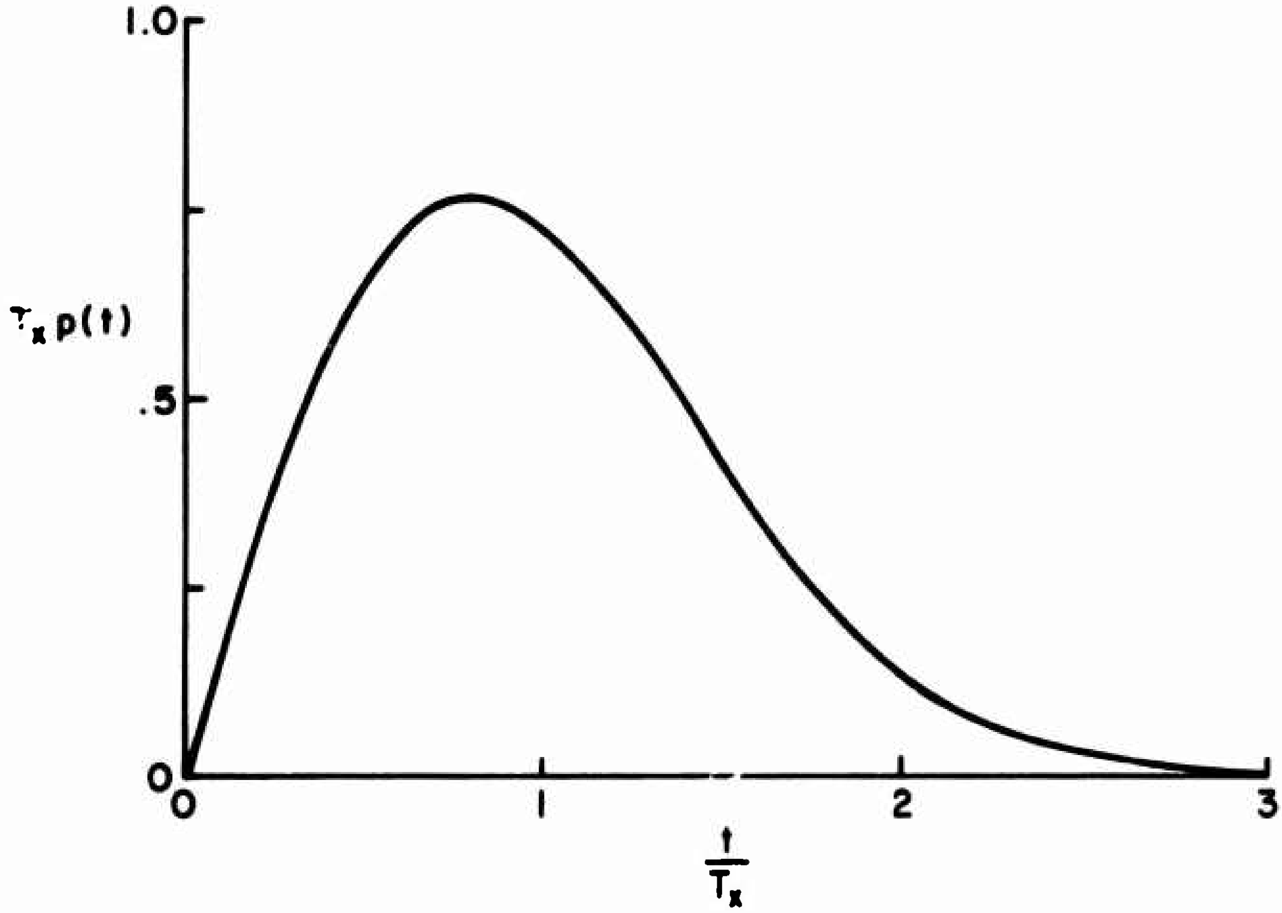

Fig. 3. Rayleigh Distribution Function for Repeat Time

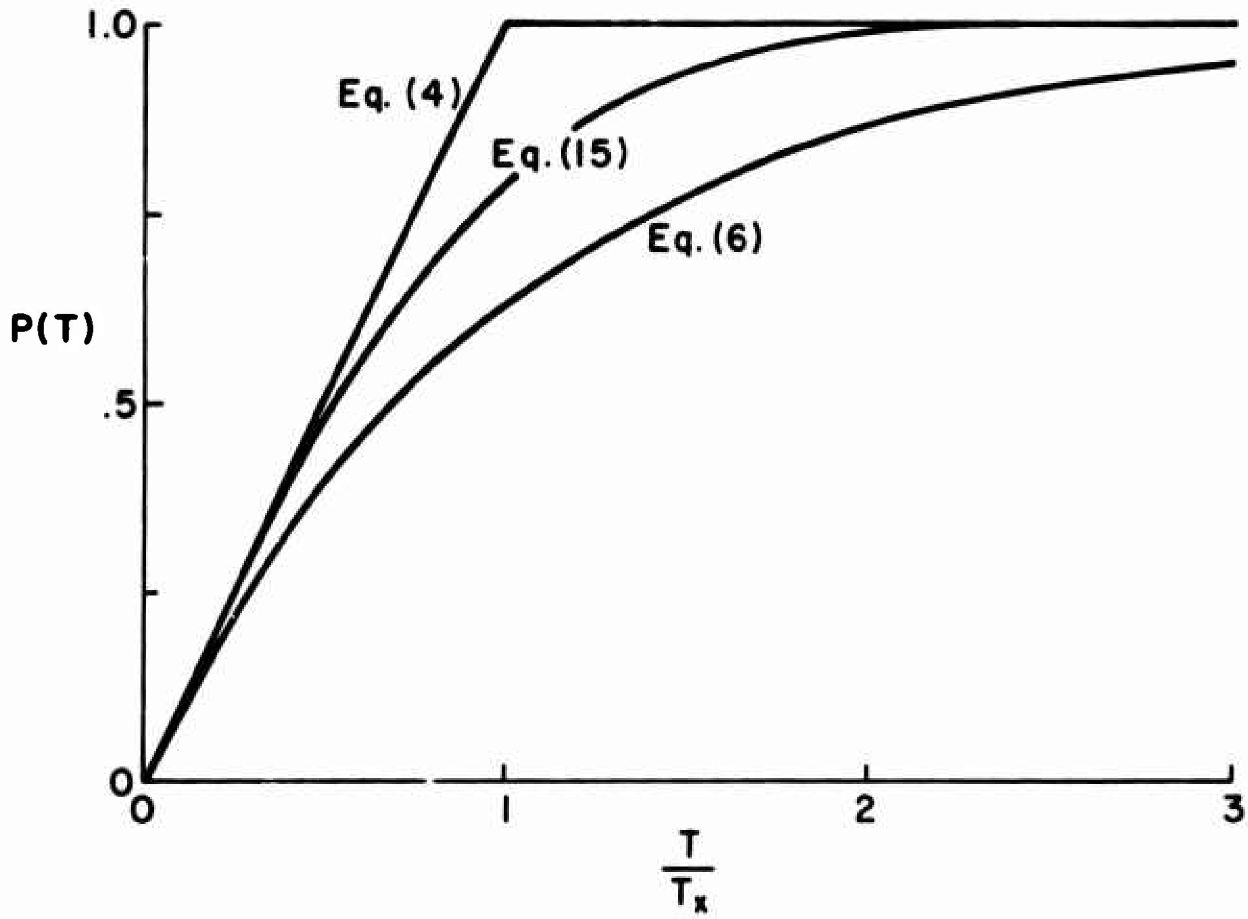

Fig. 4. Probability of Exceeding for Rayleigh Distribution of Repeat Time

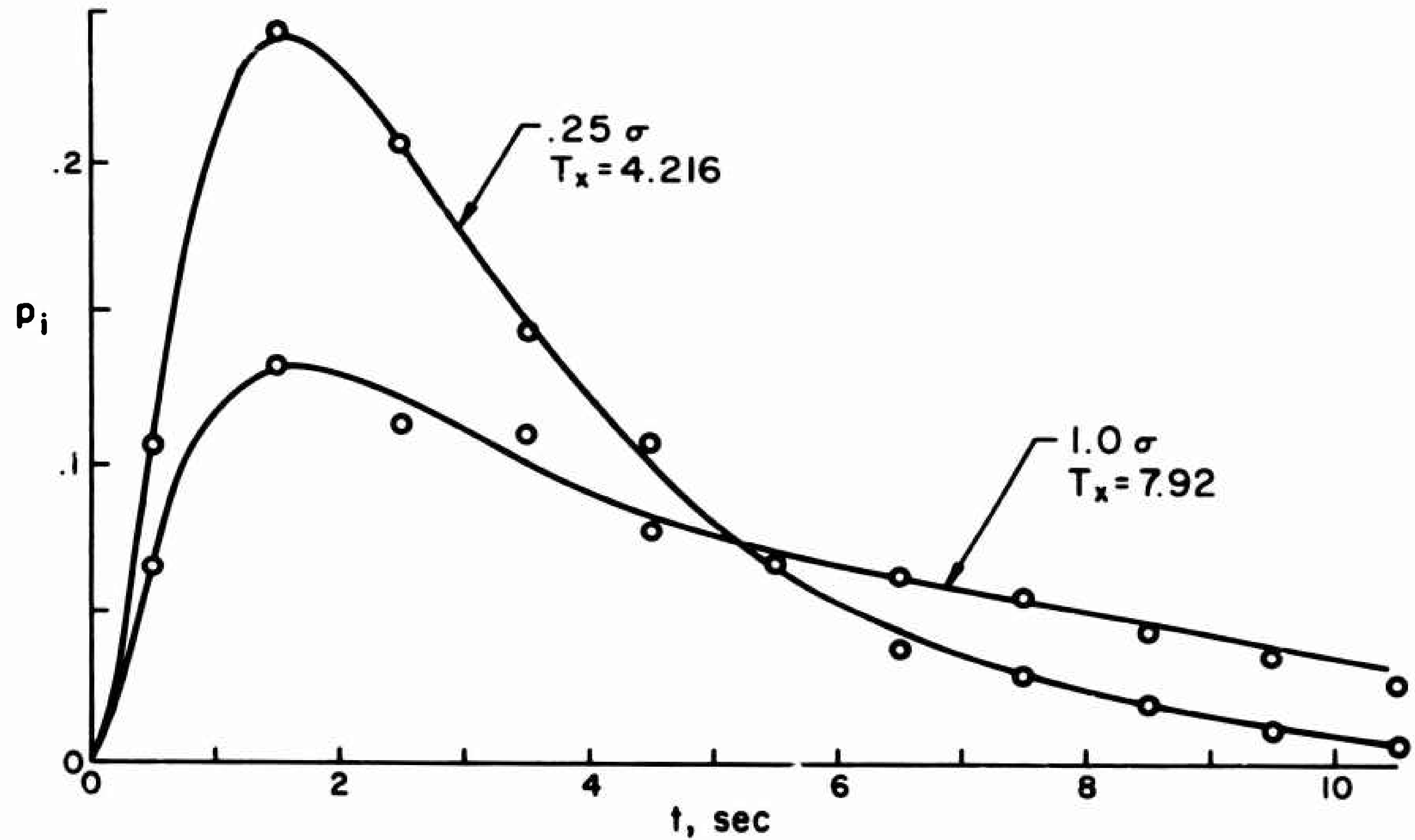

Fig. 5. Distribution Functions for Repeat Time as Obtained from a Random (Gaussian) Function

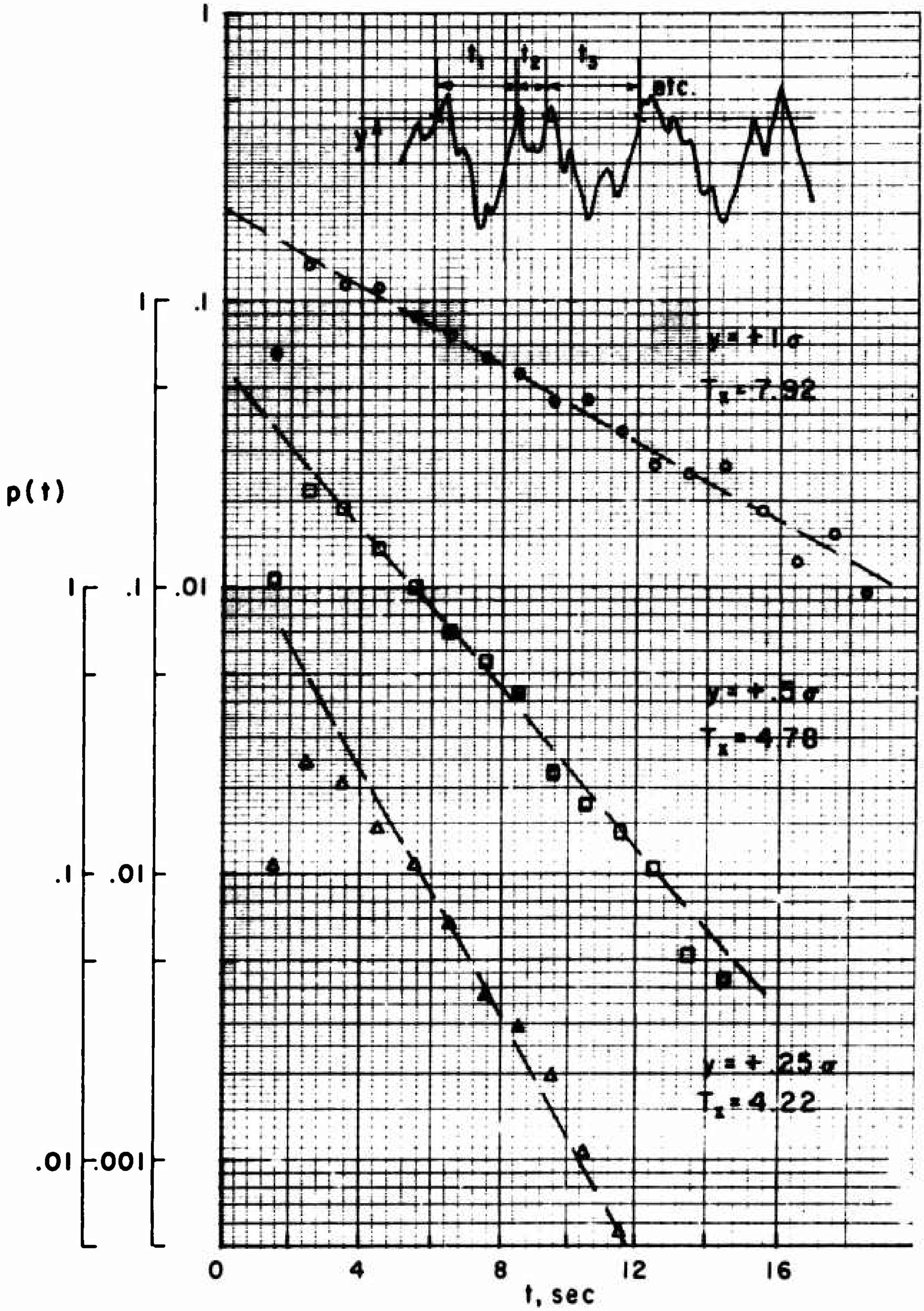

Fig. 6. Semilog Plot of Distribution Function for Repeat Time for a Random Function

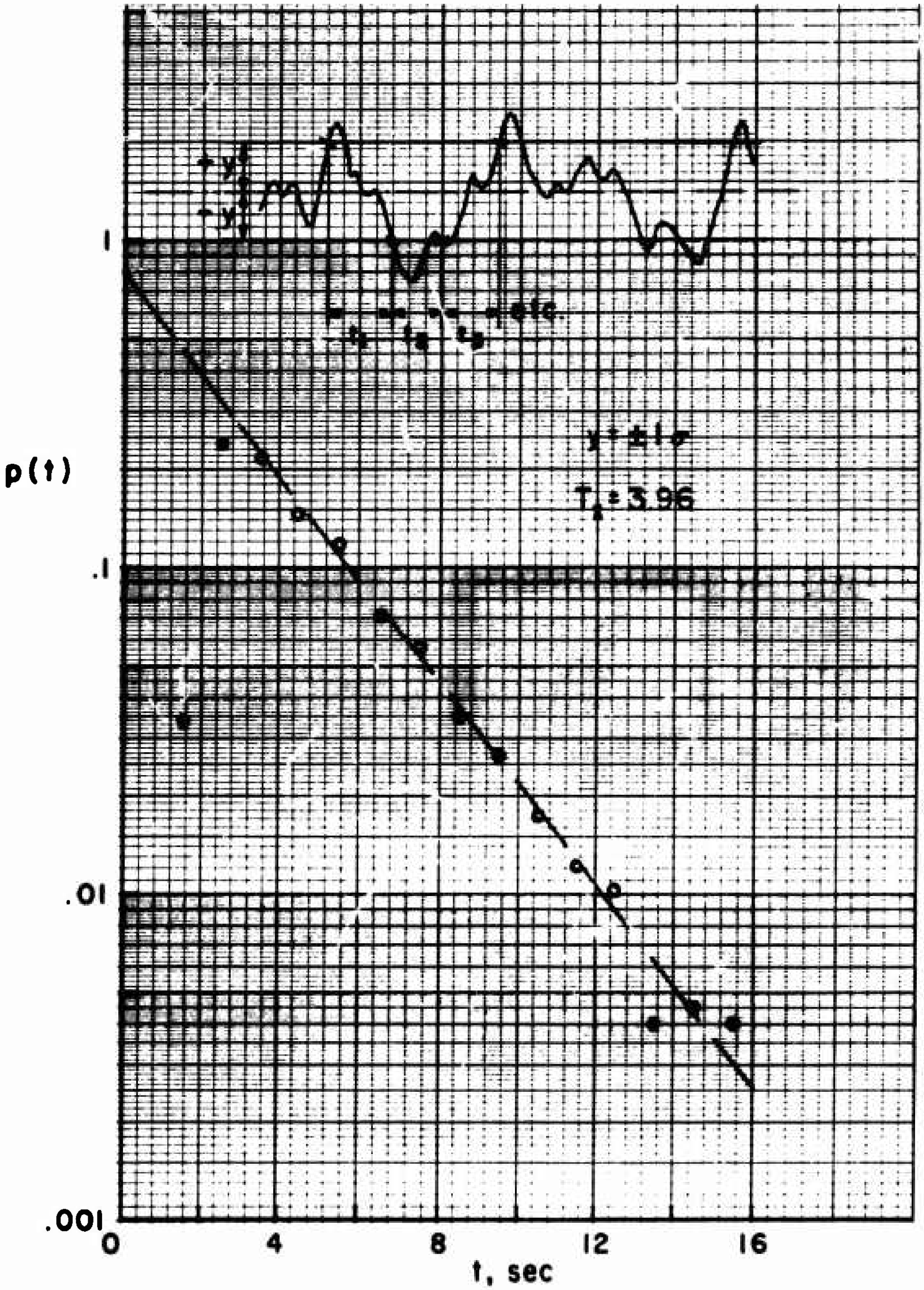

Fig. 7. Distribution Function for Repeat Time of Either Positive or Negative Values for a Random Function

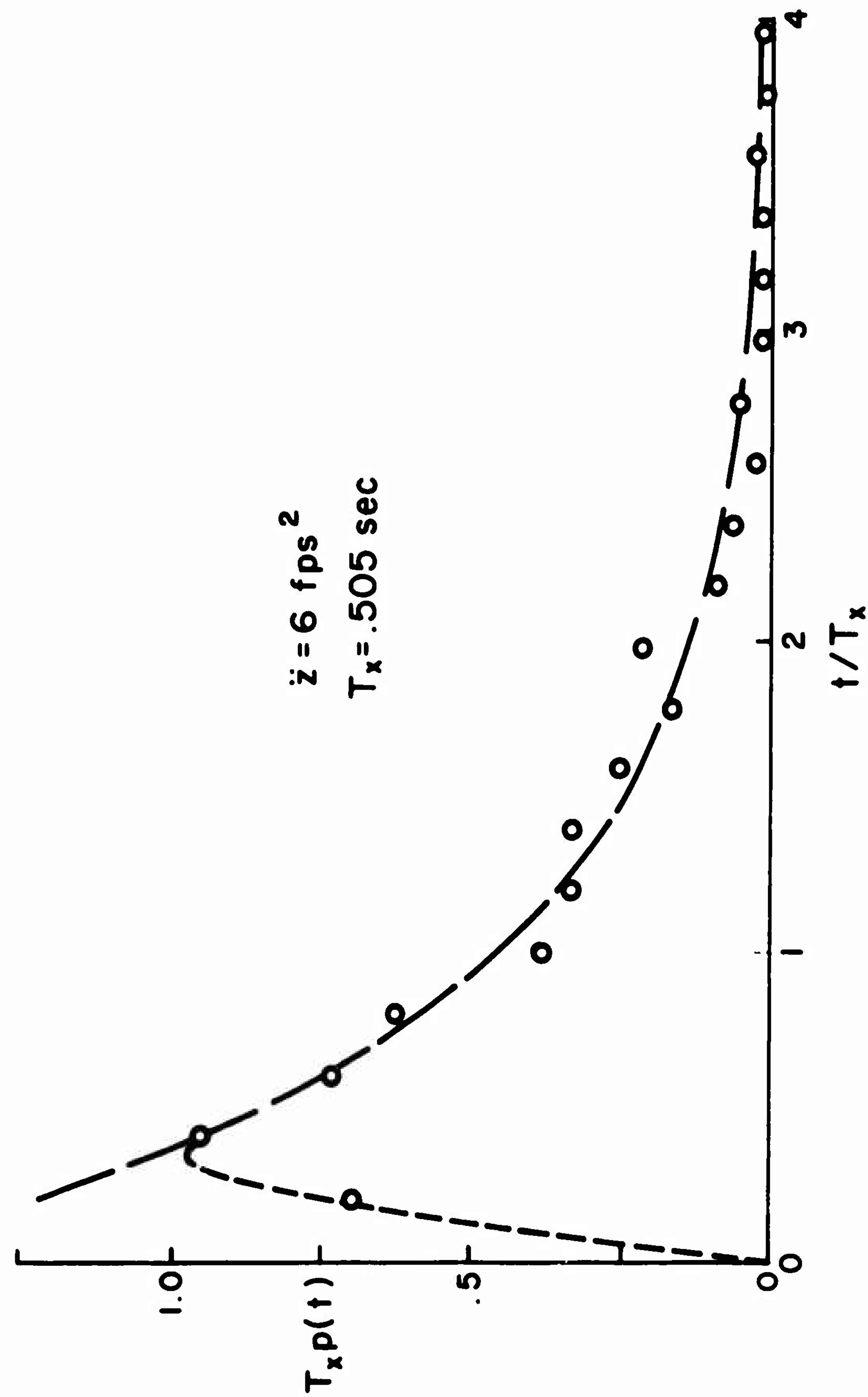

Fig. 8. Distribution Function for Repeat Time for Airplane Vertical Acceleration due to Gust Encounter

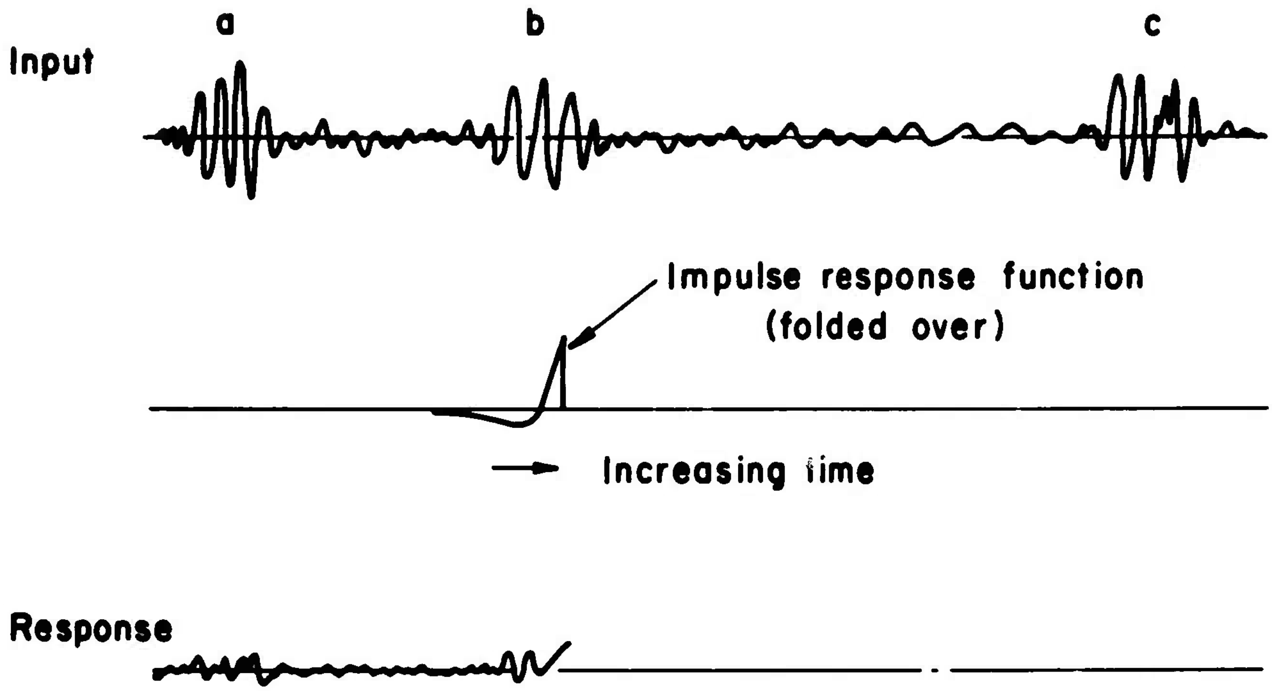

Fig. 9. System Response due to a Random Input

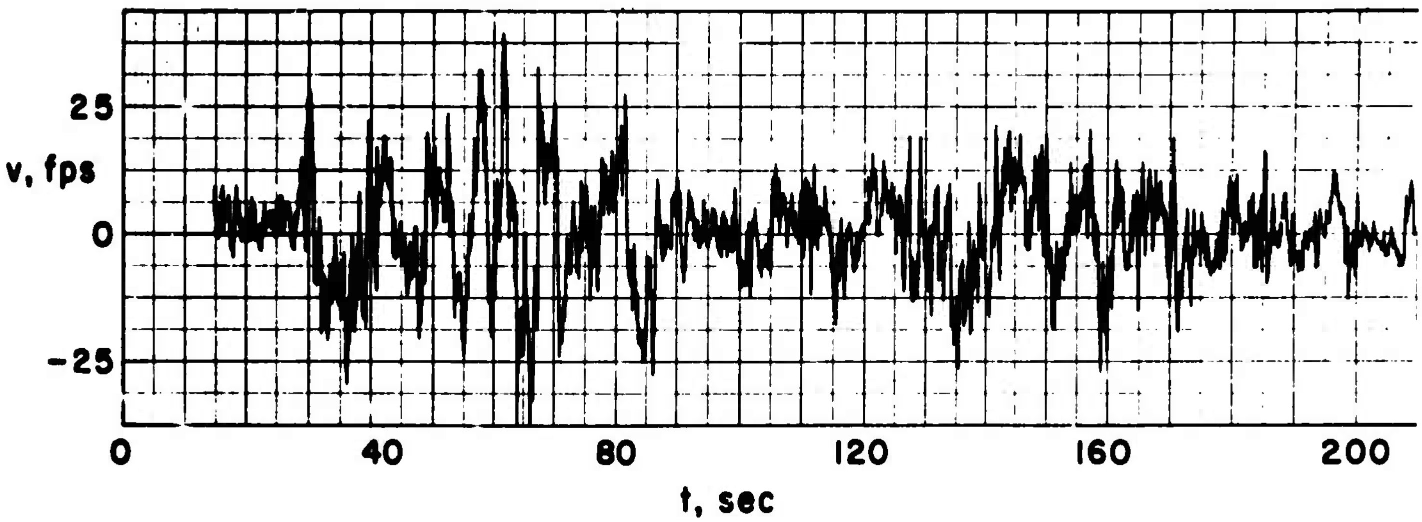

Fig. 10. Time History of Vertical Gust Velocities

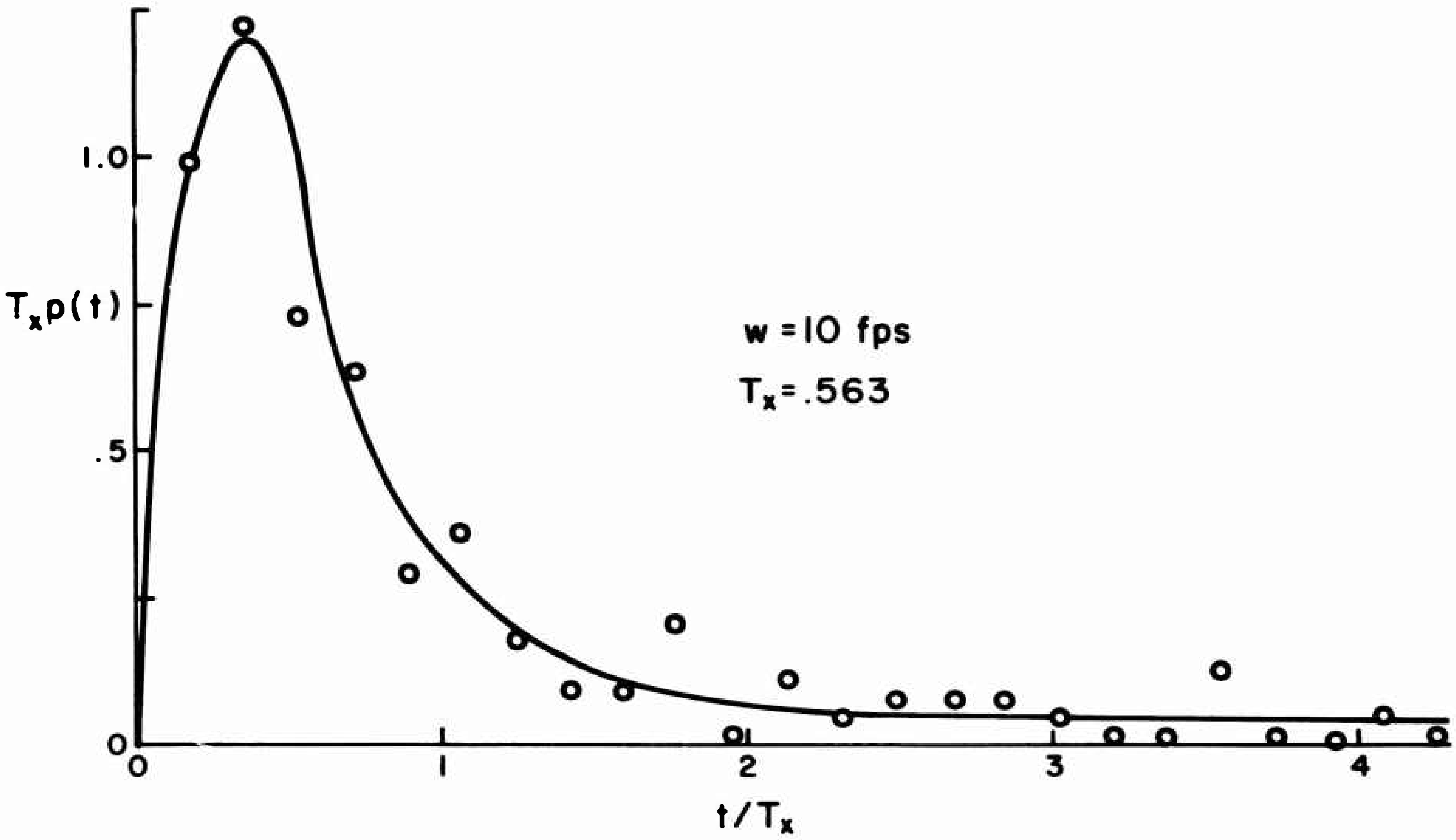

Fig. 11. Distribution Function for Repeat Time for Time History

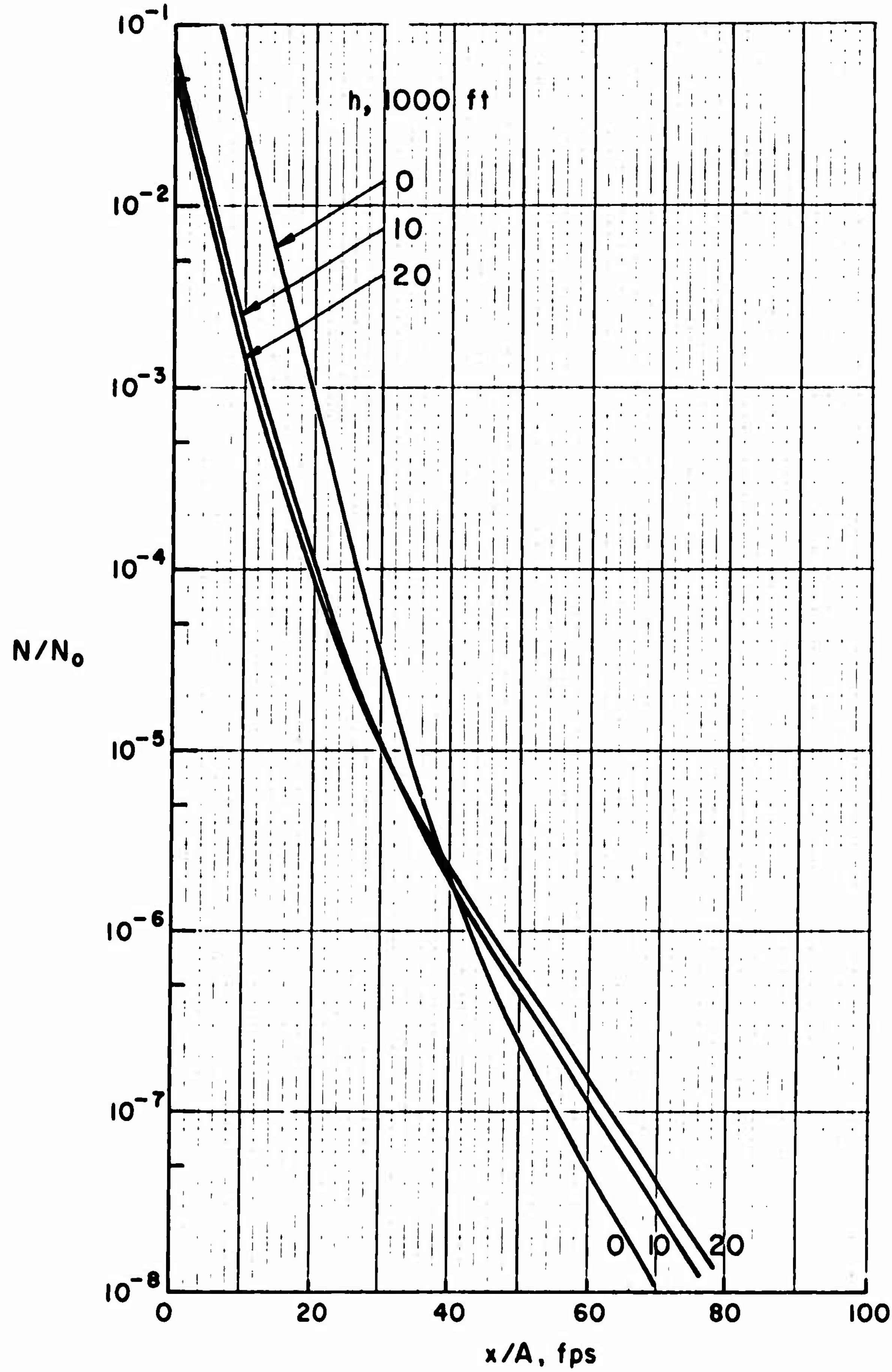

Figure 12. The updated generalized exceedance curves